System of equations with quadratics

1. Determine the coordinates of the intersection points of the parabola $y = \frac{1}{4}x^2$ and the given line.

   (a) $l_1 :\ y = \frac{1}{2}x$

   (b) $l_2 :\ y = -\frac{1}{2}x + 2$

2. How many points of intersection of the graphs of functions f and g have both coordinates integers?

   (a) $f(x) = -x + 2, \quad g(x) = x^2 - 2x$

   (b) $f(x) = x - 6, \quad g(x) = \frac{1}{4}x^2 - 2x - 3$

   (c) $f(x) = 2x - 4, \quad g(x) = 2x^2$

   (d) $f(x) = \frac{1}{2}x + 1, \quad g(x) = \frac{1}{2}x^2$

3. Solve the system of equations and give its geometric interpretation.

   (a) $\begin{cases} x + y - 4 = 0 \\ y = x^2 - 4x + 4 \end{cases}$

   (b) $\begin{cases} x + \dfrac{1}{2}y + 1 = 0 \\ y = x^2 + 2x - 2 \end{cases}$

   (c) $\begin{cases} x^2 - 4x - 2y = 0 \\ 2x + y - 2 = 0 \end{cases}$

   (d) $\begin{cases} x^2 + 4y - 16 = 0 \\ x - 2y + 4 = 0 \end{cases}$

   (e) $\begin{cases} y = 2x^2 - 4x - 3 \\ 8x + y + 5 = 0 \end{cases}$

   (f) $\begin{cases} y = 2x^2 + 8x + 5 \\ x - \dfrac{1}{4}y - \dfrac{1}{4} = 0 \end{cases}$

4. The graphs of functions f and g intersect at points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, where $x_1 < x_2$. Determine the vector $\overrightarrow{P_1P_2}$.

   (a) $f(x) = x + 1, \quad g(x) = x^2 + 1$

   (b) $f(x) = x + 1, \quad g(x) = -2x^2 + 3$

   (c) $f(x) = -2x + 3, \quad g(x) = x^2$

   (d) $f(x) = -x - 2, \quad g(x) = \frac{1}{3}x^2 - 2$

5. Mark in the coordinate plane the region described by each system of inequalities.

   (a) $\begin{cases} y > x^2 - 2 \\ y < 2x + 1 \end{cases}$

   (b) $\begin{cases} y > x - 1 \\ y \leq -x^2 + 1 \end{cases}$

   (c) $\begin{cases} y < -\dfrac{2}{3}x + \dfrac{4}{3} \\ y \geq 0.5x^2 - 2 \end{cases}$