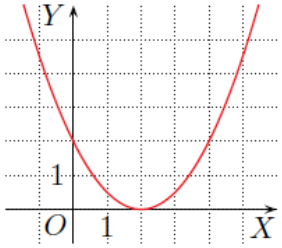
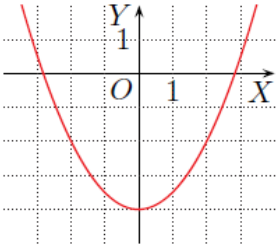
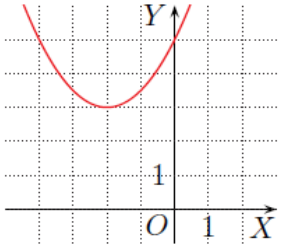
Quadratic function - revision class 1

1. Write the equation of the parabola, if the graph is a result of shifting the graph $y = \frac{1}{2}x^2$.

a) 

b) 

c) 

2. Write the standard form of the given quadratic function. Sketch the graph and read the range of the function f.

   (a) $f(x) = x^2 + 8x + 16$
   (b) $f(x) = x^2 - 4x$
   (c) $f(x) = -x^2 - 6x - 5$
   (d) $f(x) = -2x^2 + 4x - 2$

3. A Point W is the vertex of the function $f(x) = x^2 + bx + c$. Find values of b and c.

   (a) $W(0, 0)$
   (b) $W(-1, 0)$
   (c) $W(0, -1)$
   (d) $W(1, 4)$
   (e) $W(2, 3)$
   (f) $W(-1.4)$
   (g) $W(4, 1)$
   (h) $W(-1, -1)$

4. Find intervals in which the function f decreases and increases.

   (a) $f(x) = x^2 + 4\sqrt{2}x - 2$
   (b) $f(x) = -10x^2 + 200x + 3000$

5. Given the range of the function $f(x) = x^2 + bx + 1$. Find the value of b.

   (a) $\langle -3; +\infty)$
   (b) $\langle 0; +\infty)$
   (c) $\langle -8; +\infty)$

6. Points A and B belong to the graph of the function f that is symmetric with the respect to the line $x = 1$. Write a standard form of the function f and coordinates of the vertex.

   (a) $A(-1, 0)$, $B(0, 6)$
   (b) $A(0, 0)$, $B(3, 6)$

7. The parabola $y = x^2 + 2x + 1$ is shifted by a vector $\vec{u}$. Write the equation and sketch the graph of shifted parabola.

   (a) $\vec{u} = [3, 0]$
   (b) $\vec{u} = [0, -2]$
   (c) $\vec{u} = [3, -2]$
   (d) $\vec{u} = [-1, -2]$

8. What are coordinates of the vector $\vec{v}$, knowing that the graph of the function g is a result of shifting the parabola $y = 3x^2$ by the vector $\vec{v}$?

   (a) $g(x) = 3x^2 + 6x + 5$
   (b) $g(x) = 3x^2 - 6x + 5$
   (c) $g(x) = 3x^2 + 3x + 1$
   (d) $g(x) = 3x^2 + 12x + 12$

9. Solve following equations.

   (a) $x(2x + 1) = 0$
   (b) $3x^2 - 6x = 0$
   (c) $25x^2 - 1 = 0$
   (d) $x^2 - 3 = 0$
   (e) $x^2 - 6x + 9 = 0$
   (f) $4x^2 + 4x + 1 = 0$

10. Does the equation have a solution?

    (a) $x^2 - 3x + \sqrt{5} = 0$ (b) $x^2 + 4x + 3\sqrt{2} = 0$ (c) $2x^2 - 2\sqrt{6}x + 3 = 0$

11. Find zeros, write the factored form and sketch the graph of the function f. Write intervals in which the function f takes positive and negative values.

    (a) $f(x) = -x^2 + 3x + 4$
    (b) $f(x) = 3x^2 - 12x + 9$
    (c) $f(x) = x^2 - 6x + 9$
    (d) $f(x) = -2x^2 + 4x - 2$

12. Find the common points of the given parabola and the given line.

    (a) $y = -6x^2 + 7x + 6$, $y = 1 - 6x$ (b) $y = 3x^2 - 2x + 7$. $y = 4x - 4$

13. Find the set of integer numbers that satisfy the following inequality.

    (a) $x^2 \leqslant 16$ (b) $9 - x^2 > 0$ (c) $x^2 < 4x$ (d) $16 + x^2 \leqslant 8x$

14. Given functions $f(x) = 2x^2 - x + 3$ and $g(x) = x + 7$. Solve inequalities.

    (a) $f(x) \leqslant g(x)$ (b) $f(x) + x \cdot g(x) \leqslant 0$ (c) $f(x) + g(x) > 0$

15. Find a domain of the function f.

    (a) $f(x) = \frac{x+1}{(x+2)(x-1)}$ (b) $f(x) = \sqrt{6 - 5x - 6x^2}$ (c) $f(x) = \frac{x-1}{\sqrt{x^2-8x+15}}$

16. Given a function $f(x) = a(x - 7)(x + 1)$. The minimum of f is equal to -4. Find the value of the coefficient a and coordinates of the vertex.

17. The vertex of a parabola is $V = (1, 2)$. Find the equation of the parabola if:

    (a) one of x - intercept is 0.
    (b) a point $A(3, 4)$ belongs to the parabola.

18. Find value of c of the function $f(x) = x^2 + 5x + c$, if:

    (a) a point $P(-2, 1)$ belongs to the graph of function f.
    (b) the function f has only one zero.
    (c) the lowest value of f is equal to $-\frac{5}{4}$

19. Find values of b and c of the trinomial $y = x^2 + bx + c$, if:

    (a) the trinomial has two zeros 2 and -5.
    (b) the lowest value equals 7 for $x = -1$.
    (c) the trinomial takes negative values for $x \in (-1; 4)$
    (d) the graph of the parabola is symmetric about the line $x = 3$ and a y - intercept is eqal to 5.

Answers

1. (a) $y = \frac{1}{2}(x-2)^2$ (b) $y = \frac{1}{2}x^2 - 4$ (c) $y = \frac{1}{2}(x+2)^2 + 3$

2. (a) $f(x) = (x+4)^2$, $f(D) = \langle 0, +\infty)$
   (b) $f(x) = (x-2)^2 - 4$, $f(D) = \langle -4, +\infty)$
   (c) $f(x) = -(x+3)^2 + 4$, $f(D) = (-\infty; 4\rangle$
   (d) $f(x) = -2(x-1)^2$, $f(D) = (-\infty; 0\rangle$

3. (a) $b = c = 0$, $f(D) = \langle 0, +\infty)$
   (b) $b = 2, c = 1$, $f(D) = \langle 0, +\infty)$
   (c) $b = 0, c = -1$, $f(D) = \langle -1, +\infty)$
   (d) $b = -2, c = 5$, $f(D) = \langle 4, +\infty)$
   (e) $b = -4, c = 7$, $f(D) = \langle 3, +\infty)$
   (f) $b = 2, c = 5$, $f(D) = \langle 4 + \infty)$
   (g) $b = -8, c = 17$, $f(D) = \langle 1, +\infty)$
   (h) $b = 2, c = 0$, $f(D) = \langle -1, +\infty)$

4. (a) $f \searrow$ in $(-\infty; -2\sqrt{2}\rangle$, $f \nearrow$ in $\langle -2\sqrt{2}; +\infty)$
   (b) $f \nearrow$ in $(-\infty; 10\rangle$, $f \searrow$ in $\langle 10; +\infty)$

5. (a) $b = -4$ or $b = 4$ (b) $b = -2$ or $b = 2$ (c) $b = -6$ or $b = 6$

6. (a) $f(x) = -2(x-1)^2 + 8$, $W(1, 8)$ (b) $f(x) = 2(x-1)^2 - 2$, $W(1, -2)$

7. (a) $y = x^2 - 4x + 4$
   (b) $y = x^2 + 2x - 1$
   (c) $y = x^2 - 4x + 2$
   (d) $y = x^2 + 4x + 2$

8. (a) $y = 3(x+1)^2 + 2$, $\vec{v} = [-1, 2]$
   (b) $y = 3(x-1)^2 + 2$, $\vec{v} = [1, 2]$
   (c) $y = 3\left(x + \frac{1}{2}\right)^2 + \frac{1}{4}$, $\vec{v} = \left[-\frac{1}{2}, \frac{1}{4}\right]$
   (d) $y = 3(x+2)^2$, $\vec{v} = [-2, 0]$

9. (a) $x = -\frac{1}{2}$, $x = 0$
   (b) $x = 0$, $x = 2$
   (c) $x = -\frac{1}{5}$, $x = \frac{1}{5}$
   (d) $x = -\sqrt{3}$, $x = \sqrt{3}$
   (e) $x = 3$,
   (f) $x = -\frac{1}{2}$,

10. (a) yes (b) no (c) yes

11. (a) $f(x) = -(x-4)(x+1)$, $f(x) > 0$ for $x \in (-1; 4)$, $f(x) < 0$ for $x \in (-\infty; -1) \cup (4; +\infty)$
    (b) $f(x) = 3(x-1)(x-3)$, $f(x) < 0$ for $x \in (1; 3)$, $f(x) > 0$ for $x \in (-\infty; 1) \cup (3; +\infty)$
    (c) $f(x) = (x-3)^2$, $f(x) > 0$ for $x \in \mathbb{R} - \{3\}$, the function does not take negative values,
    (d) $f(x) = -2(x-1)^2$, $f(x) < 0$ for $x \in \mathbb{R} - \{1\}$, the function does not take positive values,

12. (a) $(2\frac{1}{2}, -14)$, $(-\frac{1}{3}, 3)$ (b) there are no common points

13. (a) $-4, -3, -2, -1, 0, 1, 2, 3, 4$
    (b) $-2, -1, 0, 1, 2$
    (c) $1, 2, 3$
    (d) 4

14. (a) $x \in \langle -1; 2\rangle$ (b) $x = -1$ (c) $x \in \mathbb{R}$

15. (a) $D_f = \mathbb{R} - \{-2, 1\}$ (b) $D_f = \langle -\frac{3}{2}; \frac{2}{3}\rangle$ (c) $D_f = (-\infty; 3) \cup (5; +\infty)$

16. $a = \frac{1}{4}$, $W(3, -4)$

17. (a) $y = -2x^2 + 4x$ (b) $y = \frac{1}{2}x^2 - x + \frac{5}{2}$

18. (a) $c = 7$ (b) $c = \frac{25}{4}$ (c) $c = 5$

19. (a) $b = 3, c = -10$ (b) $b = 2, c = 8$ (c) $b = -3, c = -4$ (d) $b = -6, c = 5$